First theorems of Predicate Calculus

Michael Meyling

<module@qedeq.org>

Abstract

This module includes first proofs of predicate calculus theorems.

Specification

This document has the following specification:

Name:	predtheo1
Version:	1.00.00
Rule version:	1.00.00
Orgin:	http://www.qedeq.org/0_00_53/predtheo1_1.00.00_1.00.00.qedeq

Author of this module:

Michael Meyling	mime@qedeq.org

References

This document uses the results of the following documents:

Name:	propaxiom
Version:	1.00.00
Rule version:	1.00.00
Orgin:	propaxiom_1.00.00_1.00.00.qedeq
pdf:	propaxiom_1.00.00_1.00.00.pdf
Name:	predaxiom
Version:	1.00.00
Rule version:	1.00.00
Orgin:	predaxiom_1.00.00_1.00.00.qedeq
pdf:	predaxiom_1.00.00_1.00.00.pdf

Content

First we prove a simple implication:

Theorem 0.1 (theo1).

$$\exists x\, R(x) \ \rightarrow\ \neg\forall x\, \neg R(x)$$

Proof.

1	$\forall x\, R(x) \ \rightarrow\ R(y)$	add axiom axiom5
2	$\forall x\, \neg R(x) \ \rightarrow\ \neg R(y)$	replace $R(@S_1)$ by $\neg R(@S_1)$ in 1
3	$\neg\forall x\, \neg R(x) \ \vee\ \neg R(y)$	use abbreviation impl in 2 at occurence 1
4	$(P \ \vee\ Q) \ \rightarrow\ (Q \ \vee\ P)$	add axiom axiom3
5	$(\neg\forall x\, \neg R(x) \ \vee\ Q) \ \rightarrow\ (Q \ \vee\ \neg\forall x\, \neg R(x))$	replace P by $\neg\forall x\, \neg R(x)$ in 4
6	$(\neg\forall x\, \neg R(x) \ \vee\ \neg R(y)) \ \rightarrow\ (\neg R(y) \ \vee\ \neg\forall x\, \neg R(x))$	replace Q by $\neg R(y)$ in 5
7	$\neg R(y) \ \vee\ \neg\forall x\, \neg R(x)$	MP with 3, 6
8	$R(y) \ \rightarrow\ \neg\forall x\, \neg R(x)$	reverse abbreviation impl in 7 at occurence 1
9	$\exists y\, R(y) \ \rightarrow\ \neg\forall x\, \neg R(x)$	Particularize by y in 8
10	$\exists x\, R(x) \ \rightarrow\ \neg\forall x\, \neg R(x)$	rename y into x in 9 at occurence 1

$\square$