

First theorems of Predicate Calculus

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Abstract

This module includes first proofs of predicate calculus theorems.

Specification

This document has the following specification:

Name:	predtheo1
Version:	1.00.00
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Origin:	http://www.qedeq.org/0_00_53/predtheo1_1.00.00_1.00.00.qedeq

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References

This document uses the results of the following documents:

Name:	propaxiom
Version:	1.00.00
Rule version:	1.00.00
Origin:	propaxiom_1.00.00_1.00.00.qedeq
pdf:	propaxiom_1.00.00_1.00.00.pdf
Name:	predaxiom
Version:	1.00.00
Rule version:	1.00.00
Origin:	predaxiom_1.00.00_1.00.00.qedeq
pdf:	predaxiom_1.00.00_1.00.00.pdf

Content

First we prove a simple implication:

Theorem 0.1 (theo1).

$$\exists x R(x) \rightarrow \neg \forall x \neg R(x)$$

Proof.

1	$\forall x R(x) \rightarrow R(y)$	add axiom axiom5
2	$\forall x \neg R(x) \rightarrow \neg R(y)$	replace $R(@S_1)$ by $\neg R(@S_1)$ in 1
3	$\neg \forall x \neg R(x) \vee \neg R(y)$	use abbreviation impl in 2 at occurrence 1
4	$(P \vee Q) \rightarrow (Q \vee P)$	add axiom axiom3
5	$(\neg \forall x \neg R(x) \vee Q) \rightarrow (Q \vee \neg \forall x \neg R(x))$	replace P by $\neg \forall x \neg R(x)$ in 4
6	$(\neg \forall x \neg R(x) \vee \neg R(y)) \rightarrow (\neg R(y) \vee \neg \forall x \neg R(x))$	replace Q by $\neg R(y)$ in 5
7	$\neg R(y) \vee \neg \forall x \neg R(x)$	MP with 3, 6
8	$R(y) \rightarrow \neg \forall x \neg R(x)$	reverse abbreviation impl in 7 at occurrence 1
9	$\exists y R(y) \rightarrow \neg \forall x \neg R(x)$	Particularize by y in 8
10	$\exists x R(x) \rightarrow \neg \forall x \neg R(x)$	rename y into x in 9 at occurrence 1

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